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TECHNICAL REPORT 117-13

ON INTERNAL GRAVITY WAVES GENERATED  
BY SUBMERGED DISTURBANCES

By

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August 1968

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## NOTATION

$A$	Complex wave amplitude
$d$	Depth of submergence of wedge
$d^*$	$= dN/U$
$F_1$	Function defined by Equation [49]
$F_2$	Function defined by Equation [60]
$G$	Group velocity
$H$	Heaviside function
$G_x, G_y$	Components of group velocity in Cartesian coordinates
$G_r, G_\theta$	Components of group velocity in plane polar coordinates
$G_R, G_\theta, G_y$	Components of group velocity in cylindrical polar coordinates
$k$	Radial component of wave number in plane polar coordinates
$K$	Radial component of wave number in cylindrical polar coordinates
$l$	Half-length of wedge
$m$	Source strength
$N$	Vaisala frequency
$n$	Zero or positive integer
$q, q_1, q_2$	Source distributions
$\bar{q}, \bar{q}_1, \bar{q}_2$	Fourier transform of $q, q_1$ and $q_2$ respectively
$R$	Radial coordinate in cylindrical polar coordinates
$r$	Radial coordinate in plane polar coordinates
$s$	Parameter in Laplace transform
$t$	Time

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$U, u_1, u_2$	Horizontal velocity components
$x, y, z$	Cartesian coordinates
$y$	Also an axial coordinate in cylindrical polar coordinates
$y^*$	$= yN/U$
$\alpha, \beta, \gamma$	Cartesian components of wave number
$\theta$	Polar angle of wave number plane in plane polar coordinates
$\theta$	Polar angle of wave number space in cylindrical polar coordinates
$\tau$	Half of wedge angle
$\Phi$	Polar angle in cylindrical polar coordinates
$\phi$	Polar angle in plane polar coordinates
$\omega$	Frequency

## INTRODUCTION

In a recent paper, Wu and Mei (1967) discussed the problem of gravity waves generated by a submerged two-dimensional disturbance moving horizontally in a stratified fluid with a free surface using a linearized theory. In addition to the usual free surface wave mode, internal waves that behave asymptotically like outgoing cylindrical waves were found on the downstream side of the disturbance. Experimental evidences (Long 1955, Yih 1959), however, indicate that unattenuated waves (blocking) can also exist upstream as well as downstream of the disturbance. Since Wu and Mei have only examined the vertical component of the perturbation velocity their analyses do not rule out the possibility that a linear theory can account for the existence of the blocking phenomenon; the reason for this is that the blocking perturbations would consist essentially of horizontal motions.

This report attempts to resolve this issue within the framework of a linearized theory incorporating the Boussinesq and Oseen approximations. The fluid will be taken to be unbounded and possess a constant Vaisala frequency. The concept of group velocity (Lighthill 1960, 1967) will be used to clarify the physical basis of the solution. In addition to the two-dimensional problem, three-dimensional effects on blocking will be considered. In the last section the internal waves generated by a two-dimensional wedge moving horizontally in a stratified fluid beneath a surface of large density discontinuity is considered.

## FORMULATION OF THE PROBLEM

Consider the problem of a submerged disturbance moving with constant horizontal velocity of magnitude  $U$  in an infinite expanse of density-stratified, incompressible and inviscid fluid under gravity. The Vaisala frequency  $N$  of the fluid is assumed to be constant. Choose a Cartesian coordinate system  $(x, y)$  with origin at the center of the disturbance and oriented in such a manner that gravity points in the negative  $z$  direction and the fluid appears to be moving in the positive direction.

Let us assume that the disturbance can be represented by a source distribution  $q(x, y)$ . A large class of moving solid bodies can thus be represented, although the correspondence is not simple since the body shape depends in a complex way upon the stratification of the ambient fluid and velocity.

Within the Boussinesq and Oseen approximations the linearized equation governing the horizontal component of the perturbation velocity,  $u$ , is

$$\left[ \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right)^2 \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) + N^2 \frac{\partial^2}{\partial x^2} \right] u = \frac{\partial}{\partial x} \left[ \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right)^2 + N^2 \right] q$$

[1]

where  $t$  is the time. We focus our attention on this component of the velocity because we expect the perturbations far upstream and downstream of the disturbance, if there are any, to be essentially horizontal. For a three-dimensional disturbance the corresponding equation would be



$$\left[ \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right)^2 \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) + N^2 \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) \right] u = \left[ \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right)^2 + N^2 \right] q \quad [2]$$

where  $z$  is the third space coordinate.

It is well known that problems of the sort being formulated are not determinate and it is necessary to have a way to select that particular solution corresponding to physical reality. Following Wu and Mei (1957) we consider the corresponding transient problem in which the forcing function is switched on at  $t = 0$  and take the large time limit of the resulting time-dependent solution. The switching operation is accomplished by multiplying the source term by the Heaviside function  $H(t)$ .

#### FEATURES OF THE WAVE PATTERN

Without integrating [1] directly we can obtain certain information about its associated regular, steady wave pattern using the concept of group velocity. The regular wave pattern may be considered to be composed of plane wave components and, because the pattern is steady, only those plane waves that are stationary with respect to the disturbance qualify to be included. Packets of these waves are continuously generated

through the interaction of the disturbance with the free stream. These wave packets are then swept downstream and, at the same time, they propagate with their own group velocities.

Let us, therefore, consider the behavior of plane waves in a stratified fluid stream to select those which qualify to be included in the regular wave pattern. Two dimensional plane waves have the form

$$u = A e^{i(\alpha x + \beta y - \omega t)} \quad [3]$$

where the components of the wave number vector  $(\alpha, \beta)$  and the frequency  $\omega$  are real numbers, and the amplitude  $A$  is a complex number. They must satisfy the associated homogeneous Equation of [1]. This leads to the following dispersion relation:

$$(\omega - U\alpha)^2 (\alpha^2 + \beta^2) - N^2 \alpha^2 = 0 \quad [4]$$

which has two branches, viz.

$$\omega_1 = U\alpha - \frac{N\alpha}{\sqrt{\alpha^2 + \beta^2}} \quad [5]$$

$$\omega_2 = U\alpha + \frac{N\alpha}{\sqrt{\alpha^2 + \beta^2}} \quad [6]$$

All wave numbers corresponding to  $\omega_1 = 0$  and  $\omega_2 = 0$  represent stationary plane waves, they are

$$\alpha = 0 \quad \text{for} \quad \omega_1 = 0 \text{ and } \omega_2 = 0 \quad [7]$$

$$\sqrt{\alpha^2 + \beta^2} = \frac{N}{U} \quad \text{for} \quad \omega_1 = 0 \quad [8]$$

and are shown in Figure 1 where a double line is used to indicate the double root at  $\alpha = 0$ . The regular wave pattern will be composed solely of these stationary plane waves.

To obtain further information about the wave pattern we use the concept of group velocity. Let us consider first the system of plane waves corresponding to the double root at  $\alpha = 0$ . They have wave numbers pointing in the vertical direction and their group velocities are given by

$$G_x = - \frac{N}{|\beta|} \quad , \quad G_y = 0 \quad \text{for} \quad \omega_1 = 0 \quad [9]$$

$$G_x = + \frac{N}{|\beta|} \quad , \quad G_y = 0 \quad \text{for} \quad \omega_2 = 0 \quad [10]$$

where  $G_x$  and  $G_y$  denote components in the x and y directions respectively. Packets of these waves, therefore, propagate unattenuated in the horizontal direction. Since the wave packets

are swept downstream with velocity  $(U, 0)$ , waves corresponding to the branch  $\omega_2 = 0$  always trail behind the disturbance while those corresponding to  $\omega_1 = 0$  appear ahead of the disturbance if

$|\beta| < \frac{N}{U}$  and behind if  $|\beta| > \frac{N}{U}$ . When  $|\beta| = \frac{N}{U}$  the group velocity is equal in magnitude but opposite in direction to the free stream velocity. These wave packets, therefore, do not move away from the disturbance after their generation and thus their amplitudes grow with time. The linear theory is not adequate for describing the behavior of these wave components.

Next we consider the system of plane waves corresponding to [6] — the cylindrical system. It is convenient to introduce polar coordinates defined as

$$\left. \begin{aligned} \alpha &= k \cos \theta, & \beta &= k \sin \theta \\ x &= r \cos \phi, & y &= r \sin \phi \end{aligned} \right\} \quad [11]$$

In polar coordinates the group velocities  $(G_r, G_\theta)$  of the components of this system of waves are given by

$$G_r = 0, \quad G_\theta = U \sin \theta \quad [12]$$

Figure 2 shows the vector sum of the group velocity and the free stream velocity for various components of this system. It is seen that they all point in the direction downstream of the disturbance. Thus, this system of waves always trails behind the

disturbance. When  $\theta = \pm \frac{\pi}{2}$  the group velocity is exactly equal in magnitude but opposite in direction to the free stream velocity so that again the linear theory is invalidated.

The resulting wave, of course, also depends on the excitation produced by the disturbance. For the linear theory to be valid we need to restrict ourselves to those source distributions whose Fourier transforms vanishes at the critical points

$$\alpha = 0, \quad \beta = \pm \frac{N}{U} \quad [13]$$

Let

$$q = q_1 + q_2 \quad [14]$$

and using a bar over a function to denote its Fourier transform we have

$$\bar{q} = \bar{q}_1 + \bar{q}_2 \quad [15]$$

The above requirement is satisfied if  $\bar{q}_1$  vanishes at  $\alpha = 0$  and  $\bar{q}_2$  at  $\alpha^2 + \beta^2 - \frac{N^2}{U^2} = 0$  because the sum  $\bar{q}$  vanishes at the critical points; its values at any other point, however, is arbitrary.

This limitation of the theory is entirely due to finite amplitude effects and has nothing to do with the Boussinesq approximation. Following an argument similar to the above, one can

easily show that elimination of the Boussinesq approximation fails to remove this limitation.

#### THE REGULAR WAVE PATTERN

In the previous section we have obtained a broad picture of the regular wave pattern, here we will derive it analytically. Referring to [1] let us split  $u$  into two components  $u_1$  and  $u_2$  such that

$$\left[ \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right)^2 \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) + N^2 \frac{\partial^2}{\partial x^2} \right] u_1 = \frac{\partial}{\partial x} \left[ \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right)^2 + N^2 \right] q_1 H(t)$$

[16]

and

$$\left[ \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right)^2 \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) + N^2 \frac{\partial^2}{\partial x^2} \right] u_2 = \frac{\partial}{\partial x} \left[ \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right)^2 + N^2 \right] q_2 H(t)$$

[17]

It is noted that we have split the total source distribution into two components  $q_1$  and  $q_2$  as discussed in the previous section where  $\bar{q}_1$ , the Fourier transform of  $q_1$  vanishes at  $\alpha = 0$  and  $\bar{q}_2$  at  $\alpha^2 + \beta^2 - \frac{N^2}{U^2} = 0$ .

Using the method of Fourier-Laplace transforms we obtain the following integral representations of the steady state solutions of [16] and [17]:

$$u_1 = - \frac{1}{(2\pi)^2} \lim_{s \rightarrow 0} \iint_{-\infty}^{\infty} d\alpha d\beta \frac{\alpha [(s+i\alpha U)^2 + N^2] \bar{q}_1}{(s+i\alpha U)^2 (\alpha^2 + \beta^2) + N^2 \alpha^2} e^{i(\alpha x + \beta y)} \quad [18]$$

$$u_2 = - \frac{1}{(2\pi)^2} \lim_{s \rightarrow 0} \iint_{-\infty}^{\infty} d\alpha d\beta \frac{\alpha [(s+i\alpha U)^2 + N^2] \bar{q}_2}{(s+i\alpha U)^2 (\alpha^2 + \beta^2) + N^2 \alpha^2} e^{i(\alpha x + \beta y)} \quad [19]$$

Let us first consider  $u_1$  and rewrite the integral in polar coordinate form as follows:

$$u_1 = \frac{I_m}{2\pi^2} \lim_{s \rightarrow 0} \int_0^{\pi/2} d\theta \int_0^{\infty} dk \frac{\cos \theta [s + ikU \cos \theta]^2 + N^2 \bar{q}_1(k \cos \theta, k \sin \theta)}{(s + ikU \cos \theta)^2 + N^2 \cos^2 \theta} \times$$

$$\left( e^{ikr \cos(\theta - \phi)} + e^{ikr \cos(\theta + \phi)} \right) \quad [20]$$

Since  $\bar{q}_1$  vanishes at  $\cos \theta = 0$  the integrand has simple poles at

$$k = \frac{is}{U \cos \theta} \pm \frac{N}{U} \quad [21]$$

and only their residues contribute to the regular wave pattern. Thus, retaining only the regular wave part,  $u_1$  becomes

$$u_1 = -\frac{\text{Re}}{2\pi} \left\{ \int_0^{\pi/2} d\theta \frac{N \sin^2 \theta}{U \cos \theta} \bar{q}_1 \left( \frac{N}{U} \cos \theta, \frac{N}{U} \sin \theta \right) e^{i \frac{N}{U} r \cos(\theta-\phi)} \right. \\ \left. + \int_0^{\pi/2-\phi} d\theta \frac{N \sin^2 \theta}{U \cos \theta} \bar{q}_1 \left( \frac{N}{U} \cos \theta, \frac{N}{U} \sin \theta \right) e^{i \frac{N}{U} r \cos(\theta+\phi)} \right\} \quad [22]$$

for  $x > 0$ , and for  $x < 0$  we have

$$u_1 = -\frac{\text{Re}}{2\pi} \int_{\phi-\pi/2}^{\pi/2} d\theta \frac{N \sin^2 \theta}{U \cos \theta} \bar{q}_1 \left( \frac{N}{U} \cos \theta, \frac{N}{U} \sin \theta \right) e^{i \frac{N}{U} r \cos(\theta-\phi)} \quad [23]$$

Using the method of stationary phase we obtain the following asymptotic form for  $u_1$ :



$$u_1 \sim - H(x) \frac{\text{Re}}{\sqrt{2\pi r}} \sqrt{\frac{N}{U}} \frac{\sin^2 \phi}{\cos \phi} \bar{q}_1 \left( \frac{N}{U} \cos \phi, \frac{N}{U} \sin \phi \right) e^{i \left( \frac{N}{U} r - \frac{\pi}{4} \right)} \quad [24]$$

Thus  $u_1$  represents an outgoing cylindrical wave system trailing behind the disturbance.

Next we consider  $u_2$ . Since  $\bar{q}_2$  vanishes at  $\alpha^2 + \beta^2 - \frac{N^2}{U^2} = 0$

the contributions from the integral to the regular wave pattern come from the residues of the simple poles at

$$\alpha = i s \beta \frac{U \beta \pm N}{(\beta U + N)(\beta U - N)} \quad [25]$$

Retaining only the regular wave part,  $u_2$  becomes

$$u_2 = \frac{1}{2\pi} \text{Re} \int_0^{N/U} d\beta \frac{N \bar{q}_2(0, \beta)}{\beta U - N} e^{i \beta y} - H(x) \frac{1}{\pi} \text{Re} \int_0^{\infty} d\beta \frac{N^2 \bar{q}_2(0, \beta)}{\beta^2 U^2 - N^2} e^{i \beta y} \quad [26]$$

Therefore  $u_2$  does not depend on  $x$  except for the Heaviside function which separates fore and aft behaviour, thus it represents unattenuated waves and accounts for the phenomenon of blocking.

From [26] it is noted that disturbances whose Fourier transforms vanish at  $\alpha = 0$  cannot excite the fore and aft unattenuated wave system and, therefore, no blocking can occur. These source distributions satisfy the condition

$$\int_{-\infty}^{\infty} q(x,y) dx = 0 \quad [27]$$

that is, for all  $y$  the sources are exactly balanced by the sinks. In particular, a source distribution that is antisymmetric in  $x$  would satisfy this condition.

It is also noted that the unattenuated waves ahead of the disturbance have small wave numbers. Thus a disturbance whose vertical extent  $l$  is small compared with  $U/N$  can excite only very weak forward unattenuated waves because the Fourier transform of the disturbance will be small in the low wave number range.

By integrating  $u_2$  over vertical sections upstream and downstream of the disturbance it is noted that one half of the total discharge from the source distribution flows upstream and the other half flows downstream.

### THREE-DIMENSIONAL EFFECTS

We have seen above that fore and aft unattenuated waves can be excited by two-dimensional disturbances moving horizontally in a stratified fluid. For a disturbance having a finite spanwise extension, however, this wave system attenuates due to lateral spreading and therefore, no blocking can occur.

The equation corresponding to [4] for the three-dimensional case is

$$(\omega - U\alpha)^2 (\alpha^2 + \beta^2 + \gamma^2) - N^2 (\alpha^2 + \gamma^2) = 0 \quad [28]$$

which also has two branches, viz.

$$\omega_1 = U\alpha - N \sqrt{\frac{\alpha^2 + \gamma^2}{\alpha^2 + \beta^2 + \gamma^2}} \quad [29]$$

$$\omega_2 = U\alpha + N \sqrt{\frac{\alpha^2 + \gamma^2}{\alpha^2 + \beta^2 + \gamma^2}} \quad [30]$$

In polar cylindrical coordinates defined as

$$\left. \begin{aligned} \alpha &= K \cos \theta; \gamma = K \sin \theta; \beta = \beta \\ x &= R \cos \phi; z = R \sin \phi; y = y \end{aligned} \right\} \quad [31]$$

they are

$$\omega_1 = UK \cos \theta - N \frac{K}{\sqrt{K^2 + \beta^2}} \quad [32]$$

$$\omega_2 = UK \cos \theta + N \frac{K}{\sqrt{K^2 + \beta^2}} \quad [33]$$

Therefore, the stationary plane waves are

$$K = 0 \quad \text{for } \omega_1 = 0 \text{ and } \omega_2 = 0 \quad [34]$$

$$\sqrt{K^2 + \beta^2} \cos \theta = \frac{N}{U} \text{ for } \omega_1 = 0 \cos \theta > 0 \quad [35]$$

$$\sqrt{K^2 + \beta^2} \cos \theta = -\frac{N}{U} \text{ for } \omega_2 = 0 \cos \theta < 0 \quad [36]$$

The system of plane waves given by [34] corresponds to the wave system that causes blocking in the two-dimensional case. Our primary interest is to see the three-dimensional effects on the blocking phenomenon and thus we shall concentrate our attention on this system.

Members of this system have  $z = \text{constant}$  planes as surfaces of constant phase and the components in cylindrical coordinates of their group velocities are

$$G_R = -\frac{N}{\beta}, \quad G_\theta = 0, \quad G_y = 0 \text{ for } \omega_1 = 0 \quad [37]$$

$$G_R = +\frac{N}{\beta}, \quad G_\theta = 0, \quad G_y = 0 \text{ for } \omega_2 = 0 \quad [38]$$

It is seen that those corresponding to  $\omega_1 = 0$  have energy flowing radially inwards and are, therefore, not excited by a localized disturbance.

The wave packets, while propagating with their own group velocities are, at the same time, being washed downstream. Figure 3 shows the plane view of the position of wave packets of different  $|\beta|$  one unit of time after their generation by the disturbance which is localized at the origin of the coordinate system. Wave packets with  $|\beta| > N/U$  trail behind the disturbance and are restricted to a sector with a half included angle of  $\sin^{-1} \frac{N}{|\beta|U}$  while packets with  $|\beta| < \frac{N}{U}$  appears in front as well as behind the body.

#### THE WAVES GENERATED BY A TWO-DIMENSIONAL WEDGE

In this last section we would like to apply some of the theoretical results developed in the previous sections to a specific problem.

As we have seen, the Fourier transform of the source distribution that represents the disturbance must vanish at the critical points given by Equation [1] for, otherwise, waves that have group velocities equal but opposite to the main stream would be excited. The linear theory is not adequate for treating these particular wave components. One of the source distributions that satisfies this requirement consists of two line sources of constant strength  $m$  arranged as shown in Figure 4 where  $2l$  denote the length of the source lines and  $2d$  is the distance separating them. It will later be shown that  $d$  has to be related to  $U$  and  $N$  by

$$d = \pi(n + \frac{1}{2}) U/N \quad [39]$$

where  $n$  is zero or any positive integer. Taking

$$m = 2U \tan \tau \quad [40]$$

where  $\tau$  is supposed to be small, then according to the thin body theory, the body corresponding to each of the source lines is a wedge of angle  $2\tau$  and length  $2l$  followed by a stern of thickness  $2l \tan \tau$ . If both  $\tau$  and  $l \tan \tau$  are small then, aside from the vicinity of the stagnation point and the turning point at the shoulders, the perturbations are small and we should expect the linear theory to be reasonably adequate. Furthermore, from the governing equations and the symmetry of the disturbance, it is clear that the flow is symmetric about the  $x$ -axis and the flow in the lower half plane may be considered to be produced by a wedge moving horizontally beneath a wall or a surface of large density discontinuity.

The Fourier transform of this source distribution is

$$\bar{q} = 4m \cos \beta d \frac{\sin \alpha l}{\alpha} \quad [41]$$

At  $\alpha = 0$

$$\bar{q}(0, \beta) = 4ml \cos \beta d \quad [42]$$

therefore  $d$  has to satisfy Equation [39] in order that  $\bar{q}(0, \beta)$  vanishes at the critical points.

$\bar{q}$  can now be written as the sum of  $\bar{q}_1$  and  $\bar{q}_2$  so that  $\bar{q}_1$  vanishes at  $\alpha = 0$  and  $\bar{q}_2$  at  $\alpha^2 + \beta^2 - N^2/U^2 = 0$  as follows

$$\bar{q}_1 = \frac{4mU}{N \sin \theta} \cos \left( \frac{Nd}{U} \cos \theta \right) \sin \left( \frac{Nl}{U} \sin \theta \right) \quad [45]$$

$$\bar{q}_2 = 4m \cos \beta d \frac{\sin \alpha l}{\alpha} - \frac{4mU}{N \sin \theta} \cos \left( \frac{Nd}{U} \cos \theta \right) \sin \left( \frac{Nl}{U} \sin \theta \right) \quad [46]$$

The outgoing wave system trailing behind the wedge is obtained by substituting equation [45] into equation [24] to give

$$u_1 \sim H(x) 2m \sqrt{\frac{2U}{\pi \tau N}} \tan \phi \cos \left( \frac{Nd}{U} \cos \phi \right) \sin \left( \frac{Nl}{U} \sin \phi \right) \cos \left( \frac{Nr}{U} - \frac{\pi}{4} \right) \quad [47]$$

We shall not get into the details of this wave system beyond mentioning the fact that the wave is not singular at  $\phi = \pm \pi/2$ , i.e., along the  $y$  axis, in spite of the factor  $\tan \phi$ , because  $\cos (Nd \cos \phi/U)$  vanishes there.

To obtain the fore and aft unattenuated wave system we substitute equation [46] into equation [26]. Let us consider first the forward wake. For  $x < 0$  we have

$$u_2 = \frac{m\ell N}{\pi U} F_1 \left( \frac{yN}{U}, \frac{dN}{U} \right) \quad [48]$$

where

$$\begin{aligned} F_1 \left( \frac{yN}{U}, \frac{dN}{U} \right) &= F_1(y^*, d^*) = 2 \int_0^1 d\beta \frac{\cos \beta d^* \cos \beta y^*}{\beta - 1} \\ &= -\sin d^* \left[ \sin y^* \left\{ \text{Cin}(y^* - d^*) - \text{Cin}(y^* + d^*) \right\} \right. \\ &\quad \left. + \cos y^* \left\{ \text{Si}(y^* - d^*) - \text{Si}(y^* + d^*) \right\} \right] \end{aligned} \quad [49]$$

In Figure 5 we have plotted  $F_1$  as a function of  $y^*$  for the case where

$$d^* = \pi/2 \quad [50]$$

For  $x > 0$  we have

$$u_2 = \frac{m\ell N}{\pi U} (F_1 - \pi F_2)$$



where

$$F_2(y^*, d^*) = - \sin d^* \cos y^* [\operatorname{sgn}(d^* + y^*) + \operatorname{sgn}(d^* - y^*)]$$

[51]

In Figure 6, the expression  $(F_1 - \pi F_2)$  is plotted against  $y^*$  for  $d^*$  given by Equation [50].

It is noted that for the case where the fore and aft unattenuated perturbation profiles depend only on the volume of fluid displaced by the wedge (i.e.  $m\ell$ ) but not on the shape of the wedge, we can arbitrarily vary  $m$  and  $\ell$  respectively so long as we keep  $m\ell$  constant. The cylindrical wave system, however, would not stay the same.

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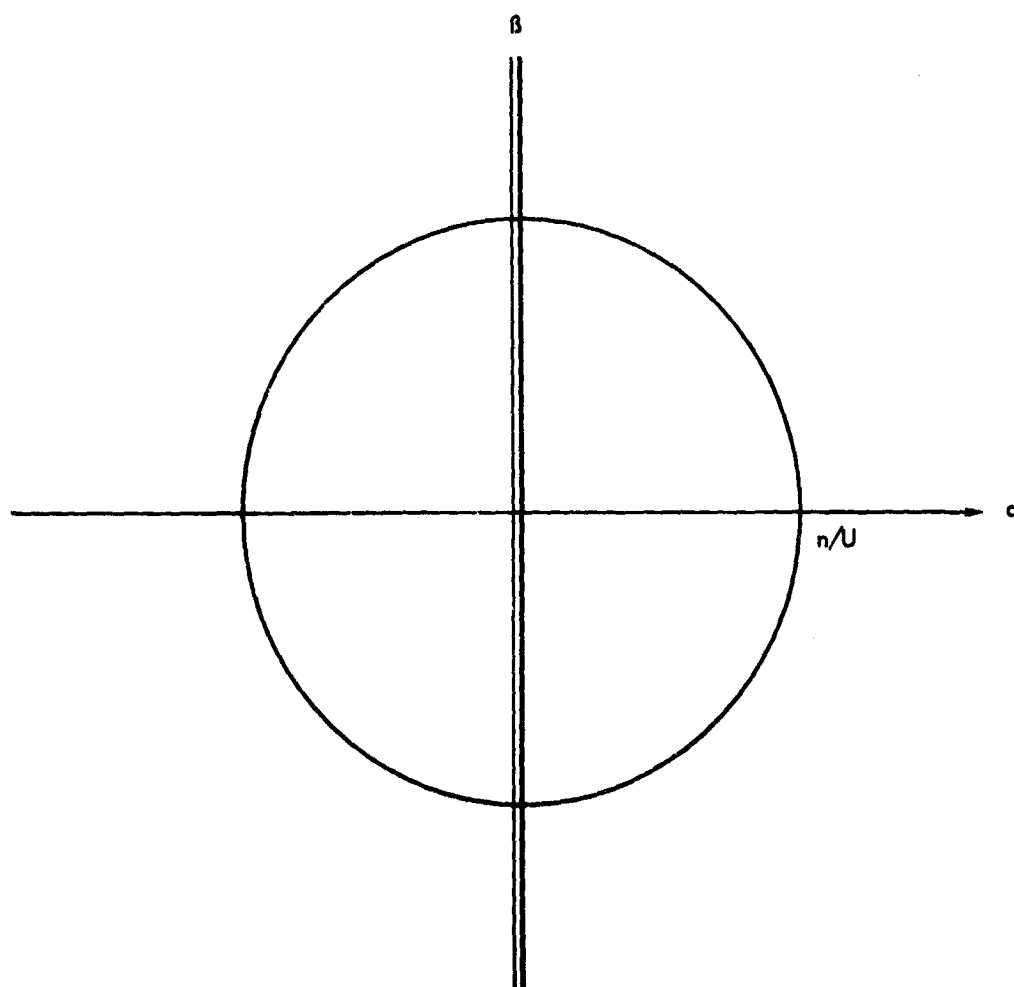


FIGURE 1 - WAVE NUMBERS OF STATIONARY PLANE WAVES  
IN A UNIFORM STREAM

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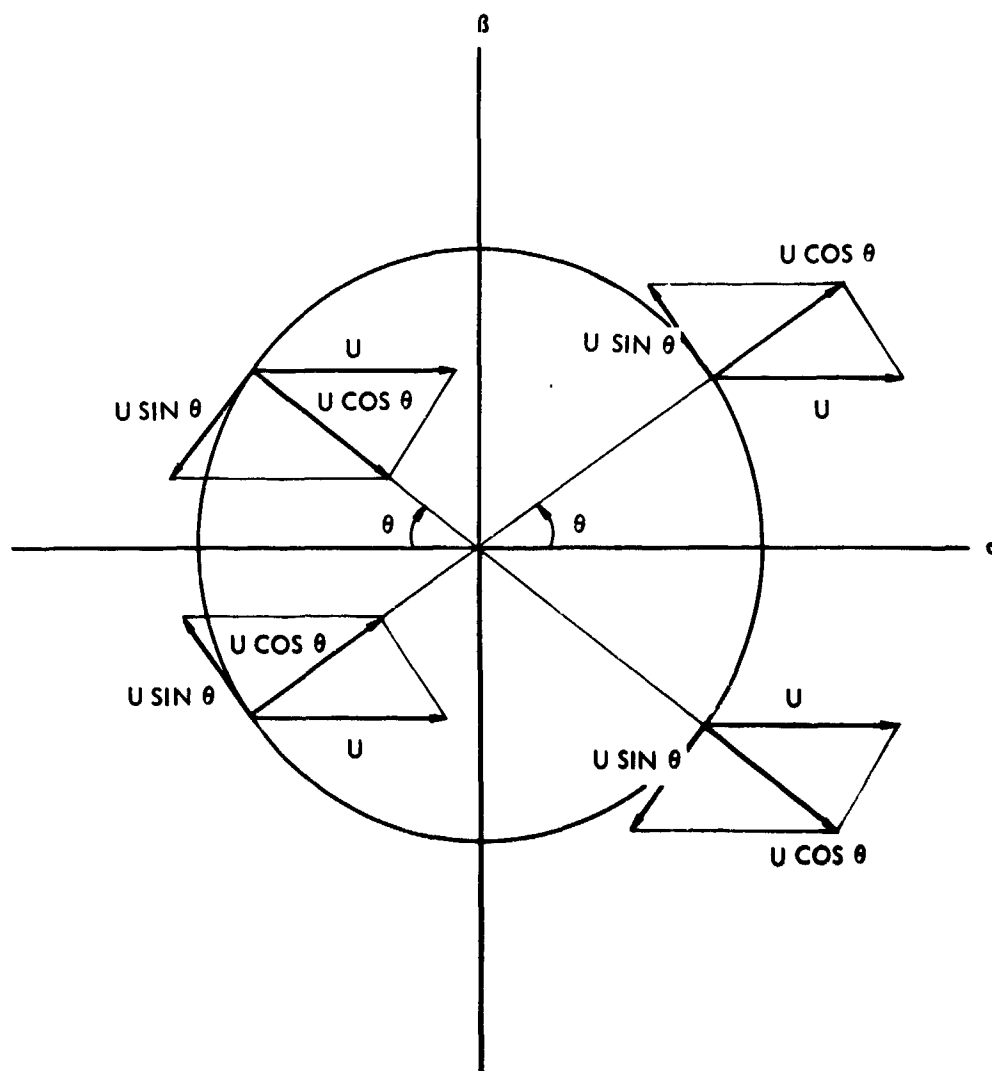


FIGURE 2 - VELOCITY OF PROPAGATION OF WAVE PACKETS OF CYLINDRICAL SYSTEM

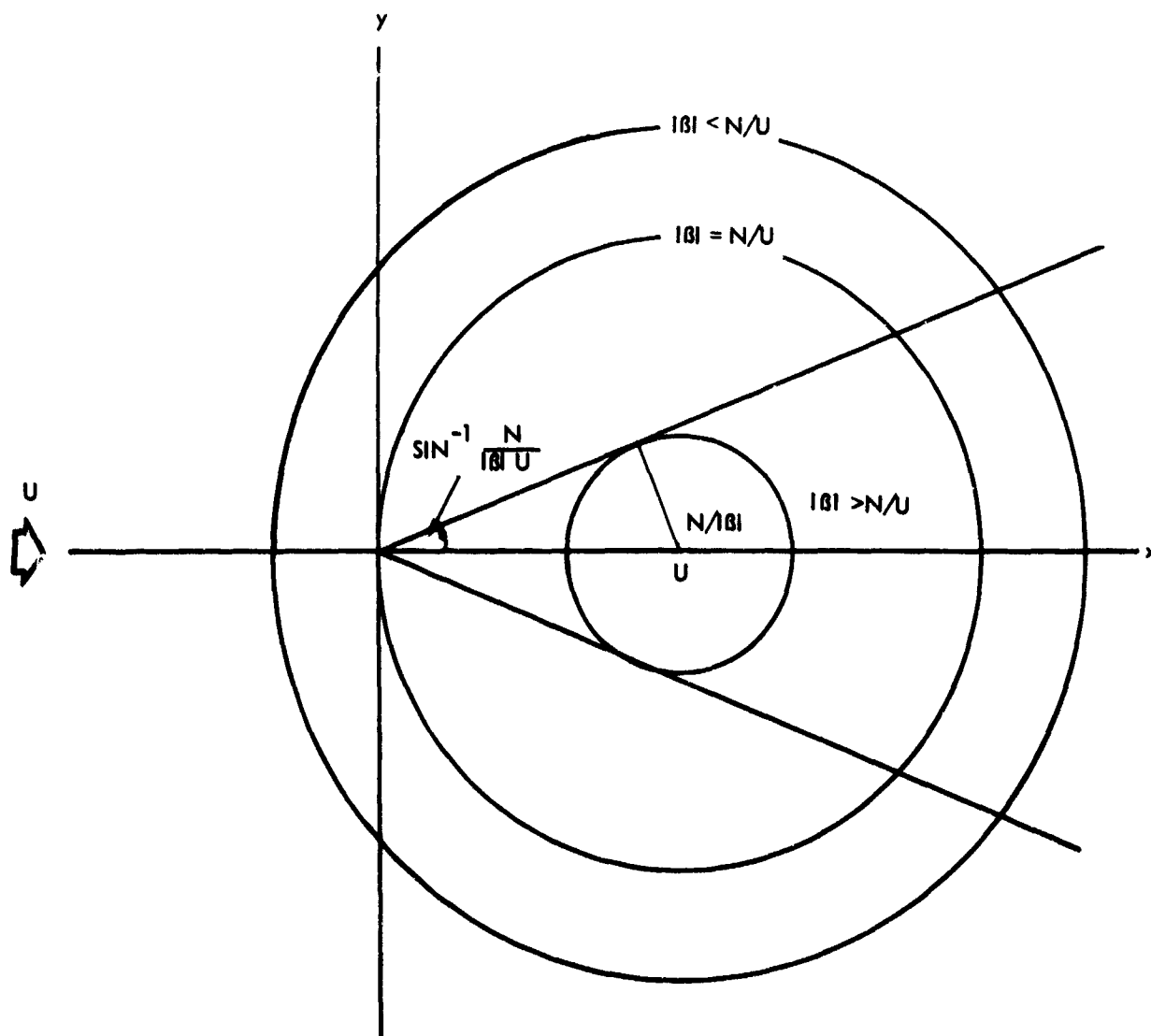


FIGURE 3 - POSITION OF WAVE PACKETS OF DIFFERENT  $|B|$  ONE UNIT OF TIME AFTER GENERATION

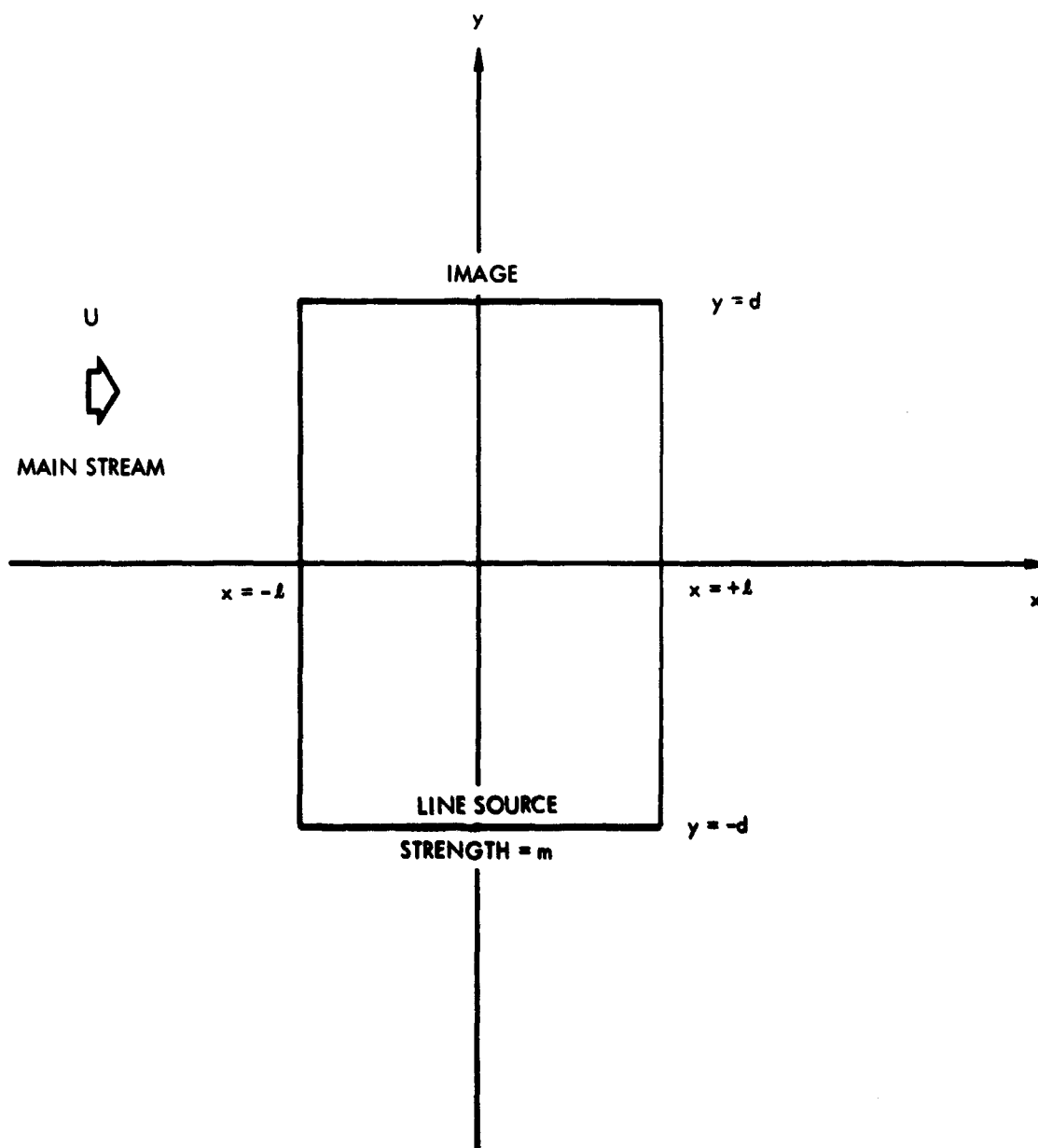


FIGURE 4 - SOURCE DISTRIBUTION REPRESENTING WEDGE BENEATH SURFACE OF LARGE DENSITY DISCONTINUITY

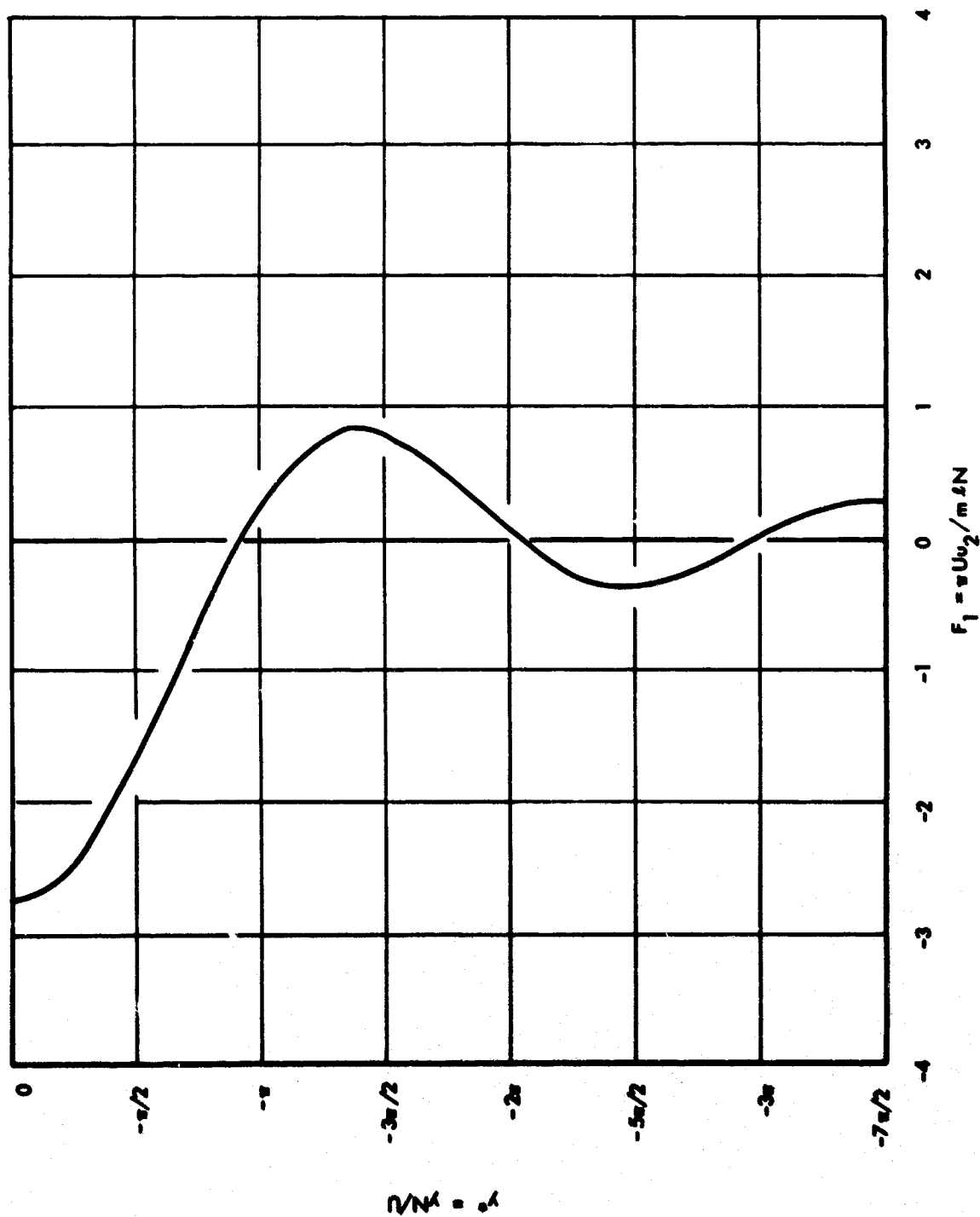


FIGURE 5 - VELOCITY PERTURBATION UPSTREAM OF WEDGE

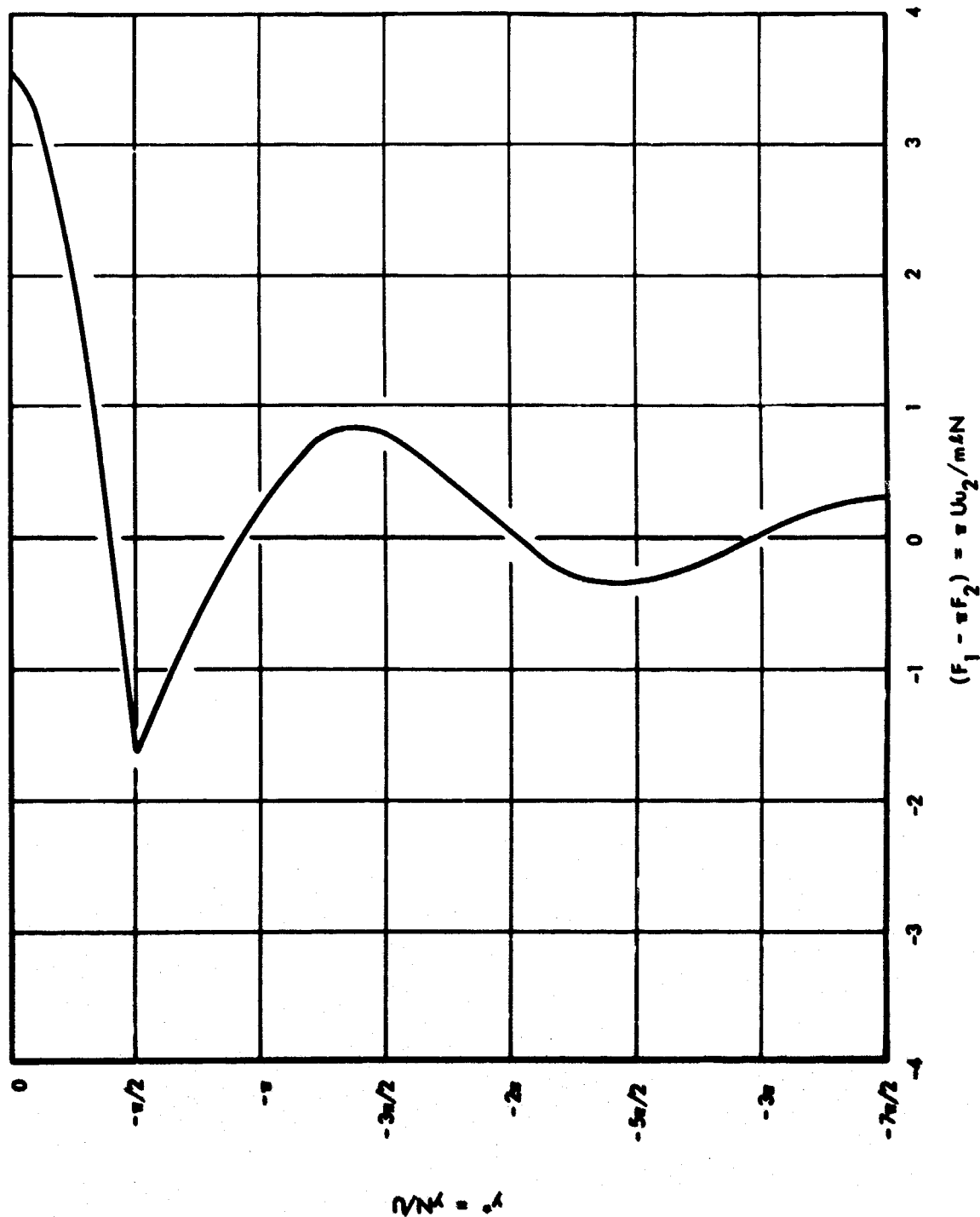


FIGURE 6 - VELOCITY PERTURBATION DOWNSTREAM OF WEDGE



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13. ABSTRACT The problem of gravity waves generated by a disturbance moving horizontally in a stratified fluid with a free surface has been considered recently by experiments and theory. The linear theory, which considered only the vertical component of the perturbation velocity, found internal waves only on the downstream side of the disturbance. The experiments, however, have disclosed the presence of upstream unattenuated waves (blocking). This report attempts to resolve this issue within the framework of a linearized theory incorporating the Boussinesq and Oseen approximations, and accounting for the blocking effects as essentially horizontal motions. The fluid is assumed unbounded with a constant Vaisala frequency, and the concept of group velocity is used to clarify the physical basis of the solution.		

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14 KEY WORDS	LINK A		LINK B		LINK C	
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